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Quantum Stirling Engine with Multiple States in One Dimensional Potential Well

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ABSTRACT

The quantum analogy for of ideal Stirling engine in one dimensional potential well has been explored. It contains single particles with multiple quantum states which is a superposition of many states. The volume in the classical engine is replaced by the length or width of the well. As in the classical one, the quantum Stirling engine consists of two isothermal processes and two isobaric processes. In the analogy of isobaric processes is constant width. The isothermal processes in the quantum engine here are defined by the constant internal energy of particles. Energy conservation is used to calculate the efficiency and work in each process. We found that the efficiency of the quantum Stirling engine has similarities with the classical Stirling engine.

Keywords: Stirling-engine, quantum stirling-engine, isothermal process, isochoric process

Introduction

Stirling engine is one heat engine that converts the color to work. Stirling engine consists of four processes, two isothermal processes, and two isobaric processes. The first process is the Isobaric process, constant volume, the pressure increase by adding the external heat. The second process is the isothermal process, constant temperature, which is work is produced. The third process is the Isobaric process which are pressure is decreased through the color is transferred to the reservoir. And the last process is isothermal (Zemansky & Dittman, 1997).

The quantum Stirling engine is a quantum analogy of the classical Stirling engine. While the working substance in classical one is gas, the quantum engine uses a single particle is placed in the quantum well. Quantum well is defined as zero potential inside well and infinite potential outside the well, so the particle freely moves inside well only. A Quantum heat engine is studied by Bender and colleagues attracted many papers to explore different aspects of it (Bender et al., 2000). For example, the relativistic heat engine was studied (Latifah & Purwanto, 2013). Many papers discuss for different potentials of quantum heat engines. In this paper, we will explore the quantum Stirling engine by considering the multiple states of a single particle.

This paper is organized as follows. First introduction and then research methods. The result and brief calculation are presented in the result. And finally, we close it with the conclusion.

Research Method

A single particle having mass m obeys the non-relativistic wave equation, the so-called Schrodinger equation. The particle is confined in a one-dimensional box of width L with infinite potential walls.

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$$-\frac{\hbar^2}{2m}\frac{\partial^2\varphi}{\partial x^2} = E\varphi$$

Or we can rewrite it

$$\frac{\partial^2\varphi}{\partial x^2} = -\frac{2mE}{\hbar^2}\varphi = -k^2\varphi$$

Or where the constant is $k^2 = \frac{2mE}{\hbar^2}$. It is related to the energy of particle $E = \frac{k^2\hbar^2}{2m}$. The solution of the equation is

$$\varphi(x) = A \sin \sin kx + B \cos \cos kx$$

The value B from the boundary condition of the wave function, $\varphi(0) = B = 0$. The other end $\varphi(L) = A \sin \sin kL = 0$ make the value

$$kL = n\pi$$

Where quantum number n=1,2,3,4, ...etc. The energy of particles now is quantized

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

The value A can be derived from normalization conditions for the wave function.

$$\varphi_n(x) = \sqrt{\frac{2}{L}} \sin \sin \frac{n\pi}{L}x$$

The absolute square of the wave function tells us the probability density of the particle for a given quantum number n. The general solution for the equation is just a linear combination for every possible solution

$$\psi(x) = \sum_{n=1}^{\infty} a_n\varphi_n(x)$$

The total energy of the system is given by

$$E = \sum_{n=1}^{\infty} |a_n|^2 E_n$$

Where $|a_n|^2$ tell us about the probability of the particle in state n and energy E_n. The Normalization condition for wave function give us

$$\sum_{n=1}^{\infty} |a_n|^2 = 1$$

The pressure in the classical process is replaced by force. The force exerted by the particle to the wall is given by:

$$F = -\frac{\partial E}{\partial L} = \sum_{n=1}^{\infty} |a_n|^2 \frac{\pi^2 \hbar^2}{mL^3}$$

As the opposite, the work of the force can be calculated by equation

$$W = \int F dL$$

The first law of thermodynamics is useful to find the relationship between internal energy, color, and work for thermodynamics processes. In a quantum engine, isothermal is defined as constant internal energy. The Isobaric process is defined by constant width.

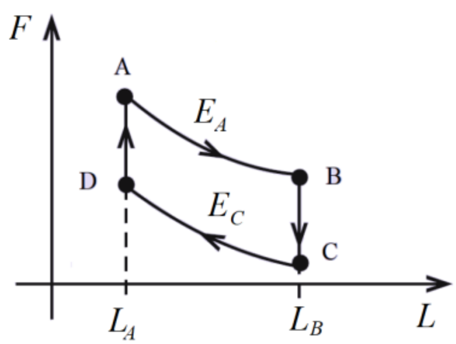

Figure 1. Stirling Engine cycle consists of four processes

In the quantum Stirling engine, we have two isothermal processes and two isochoric processes. From the figure, the process AB is isothermal where energy is constant with heat added equal to the work by the system. BC is isochoric or volume Constant force from the state. The process CD is isothermal where energy is constant with heat added equal to the work by the system. DA is isochoric or volume Constant force from the state.

Result and Discussion

The process AB is isothermal energy is constant which heat added equal to the work by the system. The transition from n=1 to the superposition state 1, 2, 3, 4,…., N. The wave function is a superposition of the sate

$$\frac{\pi^2 \hbar^2}{2mL_A^2} = E_A$$

$$\psi_{AB}(x) = \sum_{n=1}^{N} a_n \varphi_n(x)$$

With normalization condition of the wave function, $\sum_{n=1}^{N} |a_n|^2 = 1$

The energy is give by

$$E_{AB} = \sum_{n=1}^{N} |a_n|^2 n^2 \frac{\pi^2\hbar^2}{2mL^2} = E_A$$

Or

$$s_{AB}\frac{\pi^2\hbar^2}{2mL^2} = \frac{\pi^2\hbar^2}{2mL_A^2}$$

Or

$$s_{Bf}\frac{\pi^2\hbar^2}{2mL_B^2} = \frac{\pi^2\hbar^2}{2mL_A^2}$$

$$s_{Bf}L_A^2 = L_B^2$$

Where the coefficient $s_{AB} = \sum_{n=1}^{N} |a_n(L)|^2 n$, $s_{Bf} = \sum_{n=1}^{N} |a_n(L_B)|^2 n$
The force

$$F_{AB} = s_{AB}\frac{\pi^2\hbar^2}{mL^3} = \frac{2E_A}{L}$$

The change in energy is the same as the heat added to the system.

$$Q_{AB} = W_{AB} = \int_{L_A}^{L_B} F_{AB}dL = 2E_A\, ln\left(\frac{L_B}{L_A}\right)$$

Where the energy change in this process, $\Delta E_{AB} = 0$

BC is isochoric/ Isovolume

$$\psi_{BC}(x) = \sum_{n=1}^{N} a_n\varphi_n(x)$$

The constant force from the state. The energy is given by

$$E_{BC} = s_{BC}\frac{\pi^2\hbar^2}{2mL^2}$$

$s_{BC} = \sum_{n=1}^{N} |a_n(L)|^2 n$. The force can be derived from the energy,

$$F_{BC} = s_{BC}\frac{\pi^2\hbar^2}{mL^3}$$

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The work done by the system is zero W_{BC}
The energy change

$$\Delta E_{BC} = E_C - E_B = \frac{s_{Cf}\pi^2\hbar^2}{2mL_C^2} - \frac{s_{Bf}\pi^2\hbar^2}{2mL_B^2}$$

Heat energy can be derived from the law of conservation energy.

$$Q_{BC} = \Delta E_{BC} + W_{BC}$$

$$Q_{BC} = \Delta E_{BC} = E_C - E_B$$

$$= \frac{s_{Cf}\pi^2\hbar^2}{2mL_C^2} - \frac{s_{Bf}\pi^2\hbar^2}{2mL_B^2}$$

The process CD is isothermal energy is constant which heat added equal to the work by the system. The transition from to the superposition state 1, 2, 3, 4,...., N. The wave function is a superposition of the state.

$$E_C = s_{Cf}\frac{\pi^2\hbar^2}{2mL_C^2}$$

$$\psi_{CD}(x) = \sum_{n=1}^{N} \quad a_n\varphi_n(x)$$

With the normalization condition of the wave function, $\sum_{n=1}^{N} \quad |a_n|^2 = 1$

The energy is given by

$$E_{CD} = \sum_{n=1}^{N} \quad |a_n|^2 n^2 \frac{\pi^2\hbar^2}{2mL^2} = E_C$$

$$s_{CDf}\frac{\pi^2\hbar^2}{2mL^2} = s_{Cf}\frac{\pi^2\hbar^2}{2mL_C^2}$$

$$s_{Df}\frac{\pi^2\hbar^2}{2mL_D^2} = s_{Cf}\frac{\pi^2\hbar^2}{2mL_C^2}$$

$$s_{Df}L_C^2 = s_{Cf}L_D^2$$

Where the coefficient $s_{Df} = \sum_{n=1}^{N} \quad |a_n(L_D)|^2 n$

The force

$$F_{CD} = s_{CDf}\frac{\pi^2\hbar^2}{mL^3} = \frac{2E_C}{L}$$

The change in energy is the same as heat added to the system.

$$Q_{CD} = W_{CD} = \int_{L_C}^{L_D} F_{AB} dL = 2E_C \, ln\left(\frac{L_D}{L_C}\right)$$

Where the energy change in this process, $\Delta E_{CD} = 0$

DA is an isochoric, constant width

$$\psi_{DA}(x) = \sum_{n=1}^{N} a_n \varphi_n(x)$$

The energy is given by

$$E_{DA} = s_{DA} \frac{\pi^2 \hbar^2}{2mL^2}$$

$s_{DA} = \sum_{n=1}^{N} |a_n|^2 n$. The force can be derived from the energy,

$$F_{DA} = s_{DA} \frac{\pi^2 \hbar^2}{mL^3}$$

The work done to the system $W_{DA} = 0$

The energy change

$$\Delta E_{DA} = E_A - E_D = \frac{\pi^2 \hbar^2}{2mL_A^2} - \frac{s_{Df} \pi^2 \hbar^2}{2mL_D^2}$$

Heat energy can be derived from the law of conservation energy.

$$Q_{DA} = \Delta E_{DA} + W_{DA} = \frac{\pi^2 \hbar^2}{2mL_A^2} - \frac{s_{Df} \pi^2 \hbar^2}{2mL_D^2}$$

The network for one cycle is just the sum of the work for each process.

$$W_{nett} = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$= 2E_A \, ln\left(\frac{L_B}{L_A}\right) + 0 + 2E_C \, ln\left(\frac{L_D}{L_C}\right) + 0$$

$$= 2E_A \, ln\left(\frac{L_B}{L_A}\right) + 2E_C \, ln\left(\frac{L_D}{L_C}\right)$$

The next heat added to the system is only at step AB

$$\eta = 1 + \frac{Q_{CD}}{Q_{AB}} = 1 + \frac{2E_C \ ln\left(\frac{L_D}{L_C}\right)}{2E_A \ ln\left(\frac{L_B}{L_A}\right)} = 1 - \frac{E_C \ ln\left(\frac{L_C}{L_D}\right)}{E_A \ ln\left(\frac{L_B}{L_A}\right)} = 1 - \frac{E_C}{E_A}$$

Where we have used relation $L_C = L_B$ and $L_D = L_A$

Conclusion

The one-dimensional quantum Stirling engine has the same efficiency formulation as the classical Stirling engine. Energy in a quantum system is quantized and while in the classical one is not quantized. The efficiency of a quantum heat engine is higher than a classical heat engine.

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